Radical Functions Review

Specific Outcome 13
Graph and analyze radical functions (limited to functions involving one radical)

Acceptable Standard
• sketch and analyze (domain, range, invariant points, x- and y-intercepts) \( y = \sqrt{f(x)} \) given the graph or equation of \( y = f(x) \)
• find the zeros of a radical function graphically and explain their relationship to the x-intercepts of the graph and the roots of an equation

Excellent Standard
• None

Achievement Indicators
13.1 Sketch the graph of the function \( y = x \), using a table of values, and state the domain and range.
13.2 Sketch the graph of the function \( y - k = a \cdot b(x - h) \) by applying transformations to the graph of the function \( y = x \), and state the domain and range.
13.3 Sketch the graph of the function \( y = f(x) \), given the graph of the function \( y = f(x) \), and explain the strategies used.
13.4 Compare the domain and range of the function \( y = f(x) \), to the domain and range of the function \( y = f(x) \), and explain why the domains and ranges may differ.
13.5 Describe the relationship between the roots of a radical equation and the x-intercepts of the graph of the corresponding radical function.
13.6 Determine, graphically, an approximate solution of a radical equation.

Notes
• Radical functions will be limited to square roots.
• Transformations of radical functions also includes sketching and analyzing the transformation of \( y = f(x) \) to \( y = \sqrt{f(x)} \). The function \( y = f(x) \) should be limited to linear or quadratic functions.

Key Concepts:
• A Radical function is one in which the variable occurs in the radicand. Radical functions have restricted domains if the index of the radical is an even number. The domain of \( y = \sqrt{f(x)} \) consists only of the values in the domain of \( f(x) \) for which \( f(x) \geq 0 \). The range of \( y = \sqrt{f(x)} \) consists of the square root of the values in the range of \( y = f(x) \) for which \( \sqrt{f(x)} \) is defined.
The basic radical function is \( y = \sqrt{x} \) and the graph is shown.

**Basic Characteristics**

- x-intercept of 0
- y-intercept of 0
- domain \( \{x|x \geq 0, x \in \mathbb{R}\} \)
- range \( \{y|y \geq 0, y \in \mathbb{R}\} \)

Just like other functions, radical functions can be transformed. The shape of transformed radical functions is very similar to the basic radical function; however the placement and orientation may be different. The effects of changing the parameters in radical functions are the same as the effects of changing parameters in other types of functions.

\[
y = \sqrt{x} \text{ becomes } y = a\sqrt{b(x-h) + k} \text{ or } y-k = a\sqrt{b(x-h)}
\]

The table below summarizes the connection between transformations, replacements for x or y, and resulting equations established for radical functions.

<table>
<thead>
<tr>
<th>Transformation</th>
<th>Replacement for x or y</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical Translation</td>
<td>( y \rightarrow y-k )</td>
<td>( y-k = \sqrt{x} ) or ( y = \sqrt{x+k} )</td>
</tr>
<tr>
<td>Horizontal Translation</td>
<td>( x \rightarrow x-h )</td>
<td>( y = \sqrt{x-h} )</td>
</tr>
<tr>
<td>Reflection in the x-axis</td>
<td>( y \rightarrow -y )</td>
<td>( -y = \sqrt{x} ) or ( y = -\sqrt{x} )</td>
</tr>
<tr>
<td>Reflection in the y-axis</td>
<td>( x \rightarrow -x )</td>
<td>( y = \sqrt{-x} )</td>
</tr>
<tr>
<td>Reflection in the line ( y = x )</td>
<td>( x \rightarrow y, y \rightarrow x )</td>
<td>( x = \sqrt{y} ) or ( y = f^{-1}(x) )</td>
</tr>
<tr>
<td>Vertical Stretch about the x-axis</td>
<td>( y \rightarrow \frac{1}{a} )</td>
<td>( \frac{1}{a}y = \sqrt{x} ) or ( y = a\sqrt{x} )</td>
</tr>
<tr>
<td>Horizontal Stretch about the y-axis</td>
<td>( x \rightarrow bx )</td>
<td>( y = \sqrt{bx} )</td>
</tr>
</tbody>
</table>

- \( a \) – vertical stretch by a factor of \( a \) about the x-axis
  - if \( a < 0 \), then the graph of \( y = \sqrt{x} \) is reflected in the x-axis

- \( b \) – horizontal stretch by a factor of \( \frac{1}{b} \) about the y-axis
  - if \( b < 0 \), then the graph of \( y = \sqrt{x} \) is reflected in the y-axis

- \( h \) – horizontal translation to the right if \( \sqrt{(x-h)} \) or to the left if \( \sqrt{(x+h)} \)

- \( k \) – vertical translation up if \( y-k = \sqrt{x} \) or down if \( y+k = \sqrt{x} \)
Examples:

1. Given the function \( f(x) = \sqrt{3x} + 1 \), write the equation of the transformed function if:
   a) \( y = -f(x) \)
   \[ y = -\sqrt{3x} - 1 \]
   b) \( y = f(-x) \)
   \[ y = \sqrt{-3x} + 1 \]

2. Explain how to transform each of the graphs of \( y = \sqrt{x} \) to obtain the graph of:
   a) \( y = -3\sqrt{4(x-2)} + 4 \)
   Vertical Stretch by a factor of 3 about the x-axis;
   Horizontal stretch by a factor of \( \frac{1}{4} \) about the y-axis;
   Reflection in the x-axis; Translation 4 units up and 2 units right.
   b) \[ y = \sqrt{-\frac{1}{2}x - 4} + 10 \]
   Horizontal stretch by a factor of 2 about the y-axis;
   Reflection in the y-axis; Translation 10 units up and 8 units left.

3. Write the equation of the radical function that results by applying each set of transformations to the graph of \( y = \sqrt{x} \).
   a) horizontal stretch about the y-axis by a factor of 4, and a horizontal translation 5 units to the left.
   \[ y = \sqrt{\frac{1}{4}(x+5)} \rightarrow y = \sqrt{\frac{1}{4}x + \frac{5}{4}} \]
   b) Reflection about the y-axis, horizontal translation 1 unit right, vertical translation 7 units down.
   \[ y + 7 = \sqrt{-x-1} \rightarrow y = \sqrt{-x+1} - 7 \]

4. What vertical stretch is applied to \( y = \sqrt{2x} + 1 \), if the transformed graph passes through \((4, 18)\)?

   Vertical stretch by a factor of 6 about the x-axis

5. As a result of the transformation of \( y = \sqrt{x} \) into the graph of \( 2y = \sqrt{x+1} \), the point \( P(64, 8) \) is transformed to \((a, b)\). Determine the coordinates of the point \((a, b)\).

   \((63, 4)\)
Key Concept: When finding equations of radical functions, to determine if there has been a vertical or horizontal stretch, always compare to the original.

6. Refer to the above graph to write the equation of each radical function in the form \( y = a\sqrt{b(x-h)} + k \)

\[
\begin{align*}
y &= 3\sqrt{x-4} + 2 \\
y &= -2\sqrt{x+3} - 4
\end{align*}
\]
Key Concept: Square Root of a Function  (graphing \( y = \sqrt{f(x)} \) from the graph of \( y = f(x) \))

- Is only defined when \( f(x) \geq 0 \) since you cannot take the square root of negative numbers.
  - When \( f(x) < 0 \), the graph of \( y = \sqrt{f(x)} \) does not exist.
- Invariant points are \((x, 0)\) and \((x, 1)\), since \(\sqrt{0} = 0\) and the \(\sqrt{1} = 1\)
- When \(0 < f(x) < 1\), the graph of \( y = \sqrt{f(x)} \) lies above the graph of \( y = f(x) \)
- When \(f(x) > 1\), the graph of \( y = \sqrt{f(x)} \) lies below the graph of \( y = f(x) \)
- The mapping of this transformation would be \((x, y) \rightarrow (x, \sqrt{y})\)
  - The values of \( \sqrt{f(x)} \) are the square roots of the values of \( f(x) \).
  - If possible, choose values of \( f(x) \) which have simple square roots.

Examples:

7. Use the graph of \( y = f(x) \) below to sketch the graph of \( y = \sqrt{f(x)} \). State the domain, range and intercepts of the image function. State the coordinates of any invariant points.

   ![Graph of \( y = f(x) \)](image1)
   ![Graph of \( y = \sqrt{f(x)} \)](image2)

   Domain: \( x \geq 2.5 \)
   Range: \( y \geq 0 \)
   Intercepts: \( x\)-int. is 2.5
               \( y\)-int is \( \sqrt{5} \)
   Invariant point(s): \( (2.5, 0) \) and \( (-2, 1) \)

   ![Graph of \( y = f(x) \)](image3)
   ![Graph of \( y = \sqrt{f(x)} \)](image4)

   Domain: \( -3 \leq x \leq 3 \)
   Range: \( 0 \leq y \leq 3 \)
   Intercepts: \( x\)-int. are \( \pm 3 \)
               \( y\)-int is 3
   Invariant point(s): \( (\pm 3, 0) \), \( (\pm \sqrt{8}, 1) \)
Key Concept:   **Solving Radical Equations Algebraically and Graphically**

**Algebraically:**

1. List any restrictions on the variable (domain) since you cannot take the square root of a negative number. (Make the radicand(s) greater than or equal to and solve)
2. Isolate the radical, and then square each side of the equation.
3. Solve for $x$.
   a) If Linear, then isolate the variable to solve.
   b) If Quadratic, make one side equal to zero and then Factor or use Quadratic Formula to solve
4. Check for extraneous roots by substituting your solution into the original equation to check if the left side of the equation equals the right side. (must check/verify)

**Graphically:**

Intersection Method:
1. Into Y1 put equation on the left side, in Y2 put equation on the right side
2. Graph
3. Find the $x$-coordinate for the point of intersection, this will be the solution

OR

X-intercept Method:
1. Bring all terms to one side of the equation (make one side $=0$) and enter it into Y1
2. Graph
3. Find the x-intercept(s) of the graph, this will be the solution.

***Remember the calculator does not give extraneous roots.

**Examples**

1. Solve algebraically and check using your calculator

\[
7 = \sqrt{12-x} + 4
\]

\[
3 = \sqrt{12-x}
\]

\[
9 = 12-x
\]

\[
x = 3
\]
2. Identify any restrictions on the variable
   a. $x = \sqrt{x + 10} + 2$
   b. $x + 2 = \sqrt{-6x - 12}$

   $x + 10 \geq 0$
   $-6x - 12 \geq 0$
   $x \geq -10$
   $-6x \geq 12$
   $x \leq -2$

Practice Test:

1. Given that the point $(2, 4x^2), x \geq 0$ is on the function $y = f(x)$, which of the following points is on $y = \sqrt{f(x)}$?

   A. $(\sqrt{x}, 4x^2)$
   B. $(x, 2x)$ (answer)
   C. $(x, 2x^2)$
   D. $(\sqrt{x}, 2x)$

2. The radical function $y = \sqrt{f(x)}$ has an x-intercept at 2. If the graph of the function is stretched horizontally by a factor of $\frac{1}{2}$ about the y-axis, what is the new x-intercept?

   A. 2
   B. 1 (answer)
   C. $\frac{1}{2}$
   D. 4

3. If $y = f(x)$ has a range of $\{y | -4 \leq y \leq 16, y \in \mathbb{R}\}$. Then the range of the function $y = \sqrt{f(x)}$ is

   A. $\{y | -4 \leq y \leq 16, y \in \mathbb{R}\}$.
   B. $\{y | 0 \leq y \leq 4, y \in \mathbb{R}\}$ (answer)
   C. $\{y | -2 \leq y \leq 4, y \in \mathbb{R}\}$.
   D. $\{y | 0 \leq y \leq 16, y \in \mathbb{R}\}$. 
4. If \( y = \sqrt{x} \) is stretched horizontally by a factor of 6, which is the resulting equation

A. \( y = \frac{1}{6} \sqrt{x} \)
B. \( y = 6 \sqrt{x} \)
C. \( y = \sqrt{\frac{1}{6} x} \)  \(\text{(answer)}\)
D. \( y = \sqrt{6x} \)

5. When solving the equation \( x - 3 = \sqrt{x-1} \) the extraneous root is

A. -2
B. 2  \(\text{(answer)}\)
C. -5
D. 5

6. If \( f(x) = x + 1 \), which point is on the graph of \( y = \sqrt{f(x)} \) ?

A. (0, 0)
B. (0, 1)  \(\text{(answer)}\)
C. (1, 0)
D. (1, 1)

7. Which function has a domain of \( \{ x \mid x \geq 5, x \in \mathbb{R} \} \) and a range of \( \{ y \mid y \geq 0, y \in \mathbb{R} \} \)?

A. \( f(x) = \sqrt{x-5} \)  \(\text{(answer)}\)
B. \( f(x) = \sqrt{x} - 5 \)
C. \( f(x) = \sqrt{x+5} \)
D. \( f(x) = \sqrt{x+5} \)
8. This graph is of the function \( y = f(x) \).

What is the graph of \( y = \sqrt{f(x)} \) ?

9. If the function \( y = \sqrt{-3(x + c)} + c \) passes through the point (-1, 1), then what is the value of \( c \)?

\( c = 1 \)
10. State the coordinate of any invariant points when \( f(x) = \frac{1}{2}x - 3 \) is transformed to \( y = \sqrt{x} \).

\((6, 0) \text{ and } (8, 1)\)

11. Determine the x-intercept of \( y = -2\sqrt{x+4} + 3 \), to the nearest hundredth.

\(x\)-int. is -1.75 \hspace{1cm} \text{(Graph)}

12. The point \((4, 2)\) is on the graph of \( f(x) = \sqrt{x} \). The graph is transformed into \( g(x) \) by a horizontal stretch by a factor of 2, a reflection about the \( x \)-axis, and a translation up 3 units. Determine the coordinates of the corresponding point on the graph of \( g(x) \).

\((8, 1)\)

13. State the invariant point(s) when \( y = x^2 - 25 \) is transformed into \( y = \sqrt{x^2 - 25} \).

\((\pm 5, 0) \text{ and } (\pm\sqrt{26}, 1)\)

14. The graph of \( f(x) = \sqrt{2x} \) is horizontally translated 6 units left. State the equation of the translated function \( g(x) \).

\[ y = \sqrt{2(x+6)} \rightarrow y = \sqrt{2x+12} \]

15. The graph of \( f(x) = \sqrt{x} \) is stretched vertically by a factor of 4, reflected in the \( y \)-axis, vertically translated up 3 units and horizontally translated left 5 units. Write the equation of the transformed function, \( g(x) \).

\[ g(x) = 4\sqrt{-(x+5)} + 3 \rightarrow g(x) = 4\sqrt{-x-5} + 3 \]

16. The solution to the equation \( 2\sqrt{x} - \sqrt{x+4} = 3 \), to the nearest tenth, is _______.

\(x = 12.4\)
Rational Functions Review

**Specific Outcome 14**
Graph and analyze rational functions (limited to numerators and denominators that are monomials, binomials and trinomials)

**Acceptable Standard**
- sketch and analyze (vertical asymptotes or point of discontinuity, domain, x- and y-intercepts) rational functions
- find the zeros of a rational function graphically and explain their relationship to the x-intercepts of the graph and the roots of an equation

**Excellent Standard**
- acceptable standard, and;
- determine the equation of a horizontal asymptote and the range of a rational function
- find the coordinates of the point of discontinuity of a rational function

**Achievement Indicators**
14.1 Graph, with or without technology, a rational function.
14.2 Analyze the graphs of a set of rational functions to identify common characteristics.
14.3 Explain the behaviour of the graph of a rational function for values of the variable near a non-permissible value.
14.4 Determine if the graph of a rational function will have an asymptote or a hole for a non-permissible value.
14.5 Match a set of rational functions to their graphs, and explain the reasoning.
14.6 Describe the relationship between the roots of a rational equation and the x-intercepts of the graph of the corresponding rational function.
14.7 Determine, graphically, an approximate solution of a rational equation.

**Notes**
- Oblique or slant asymptotes are not a part of this outcome. All graphs are restricted to horizontal and vertical asymptotes.
- Numerators and denominators should be limited to degree two or less.
- Transformations of rational functions does NOT include the transformation of \( y = f(x) \) to \( y = \frac{1}{f(x)} \).

**Key Concepts:**
- Rational functions are of the form \( f(x) = \frac{P(x)}{Q(x)} \), where \( P(x) \) and \( Q(x) \) are polynomial expressions and \( Q(x) \neq 0 \). The degree of \( Q(x) \) needs to be greater than zero or the function is simply a polynomial function.

**Examples:**
- \( y = \frac{3}{x + 2} \)
- \( y = \frac{x^2 + 5x}{x^2 + 7x + 10} \)
- \( y = \frac{2x^2 + 5x - 3}{x + 3} \)
- \( y = \frac{6}{x - 2} - 3 \)
**Vertical Asymptote (VA)**

Is the Non-Permissible Value or Restriction (from denominator = 0)

\[ x = c, \]

where \( c \) is some constant.

**Vertical Asymptote** – corresponds to a non-permissible value in the equation which is found when the denominator is set equal to zero.

\[
y = \frac{3}{x+2}
\]

has a vertical asymptote at \( x = -2 \)

\[
y = \frac{3x}{x^2+2x-8} = \frac{3x}{(x+4)(x-2)}
\]

has two vertical asymptotes at \( x = 2 \) and \( x = -4 \)

***But not all non-permissible values result in a vertical asymptote. Some non-permissible values result in a point of discontinuity (hole) in the graph.

**Point of Discontinuity** – is a point on the graph of a function that is not continuous.

- Occurs when numerator and denominator of a rational function are simplified by dividing out the common factor.
- The missing ‘single point’ is represented by an open circle on the graph. **SE**
- considered to be a “hole” in the graph

The function, \( f(x) = \frac{x^2-5x+6}{x-3} \) can be simplified in the following way (note the NPV at \( x = 3 \))

\[ f(x) = \frac{(x-2)(x-3)}{x-3} \]

\[ f(x) = x-2 \]

Which makes a linear graph with a hole at \( x = 3 \)
To determine the point of discontinuity, substitute $x = 3$ into the simplified equation. SE

\[
\begin{align*}
  f(x) &= (x - 2) \\
  f(x) &= (3) - 2 \\
  f(x) &= 1
\end{align*}
\]

The point of discontinuity occurs at $(3, 1)$

Procedure to determine coordinates of the point of discontinuity algebraically,

1. Factor the rational expression
2. Simplify the rational expression by cancelling out factors.
3. Substitute the NPV of $x$ into the simplified form.

**Examples:**

1. Complete the table:

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>$y = \frac{4}{x}$</th>
<th>$y = \frac{2x - 1}{x - 7}$</th>
<th>$y = \frac{6}{x - 2} - 3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical Asymptote</td>
<td>$x = 0$</td>
<td>$x = 7$</td>
<td>$x = 2$</td>
</tr>
<tr>
<td>Domain</td>
<td>( {x \mid x \neq 0, x \in \mathbb{R}} )</td>
<td>( {x \mid x \neq 7, x \in \mathbb{R}} )</td>
<td>( {x \mid x \neq 2, x \in \mathbb{R}} )</td>
</tr>
<tr>
<td>Horizontal Asymptote</td>
<td>$y = 0$</td>
<td>$y = 2$</td>
<td>$y = -3$</td>
</tr>
<tr>
<td>Range</td>
<td>( {y \mid y \neq 0, y \in \mathbb{R}} )</td>
<td>( {y \mid y \neq 2, y \in \mathbb{R}} )</td>
<td>( {y \mid y \neq -3, y \in \mathbb{R}} )</td>
</tr>
</tbody>
</table>

2. Complete the table:

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>$y = \frac{x^2 + 5x}{x^2 + 7x + 10}$</th>
<th>$y = \frac{x^2 - 7x + 12}{x^2 - 9}$</th>
<th>$y = \frac{2x^2 + 5x - 3}{x + 3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical Asymptote</td>
<td>$x = -2$</td>
<td>$x = -3$</td>
<td>none</td>
</tr>
<tr>
<td>Horizontal Asymptote</td>
<td>$y = 1$</td>
<td>$y = 1$</td>
<td>none</td>
</tr>
<tr>
<td>Point of Discontinuity</td>
<td>$(5, \frac{5}{3})$</td>
<td>$(3, \frac{7}{6})$</td>
<td>$(-3, -7)$</td>
</tr>
<tr>
<td>x-intercept</td>
<td>0</td>
<td>4</td>
<td>0.5</td>
</tr>
<tr>
<td>y-intercept</td>
<td>0</td>
<td>-4/3</td>
<td>-1</td>
</tr>
<tr>
<td>Domain</td>
<td>( {x \mid x \neq -3, x \in \mathbb{R}} )</td>
<td>( {x \mid x \neq \pm 3, x \in \mathbb{R}} )</td>
<td>( {x \mid x \neq -3, x \in \mathbb{R}} )</td>
</tr>
<tr>
<td>Range</td>
<td>( {y \mid y \neq \frac{5}{3}, y \in \mathbb{R}} )</td>
<td>( {y \mid y \neq \frac{7}{6}, y \in \mathbb{R}} )</td>
<td>( {y \mid y \neq -7, y \in \mathbb{R}} )</td>
</tr>
</tbody>
</table>
Practice Test

1. The x-intercept of \( y = \frac{k}{x+1} - 2 \) is 0.5. What is the value of \( k \)?
   - A. 1.0
   - B. 1.5
   - C. 2.5
   - D. 3.0 \((\text{answer})\)

2. Consider the function \( g(x) = \frac{2x}{1-x^2} \). Which statement is false?
   - A. \( g(x) \) has two vertical asymptotes.
   - B. \( g(x) \) is not defined when \( x = 0 \). \((\text{answer})\)
   - C. \( g(x) \) has one zero.
   - D. \( g(x) \) is a rational function.

3. Which of the following is true of the rational function \( y = \frac{3}{x-2} + 6 \)?
   - A. It has a zero at \( x = 2 \)
   - B. Its range is \( \{ y \mid y \in \mathbb{R} \} \)
   - C. It is equivalent to \( y = \frac{6x-9}{x-2} \) \((\text{answer})\)
   - D. It has a vertical asymptote at \( x = 6 \).

4. The graph of which function has a point of discontinuity at \( x = 1 \)?
   - A. \( y = \frac{x-1}{x^2-1} \) \((\text{answer})\)
   - B. \( y = \frac{x+1}{x^2-1} \)
   - C. \( y = \frac{x^2-1}{x+1} \)
   - D. \( y = \frac{x^2+1}{x-1} \)

5. Which function has a domain of \( \{ x \mid x \neq 1, x \in \mathbb{R} \} \) and a range of \( \{ y \mid y \neq 3, y \in \mathbb{R} \} \)?
   - A. \( y = \frac{x}{x-1} + 3 \)
   - B. \( y = \frac{3x^2-3x}{x^2-4x+3} \)
   - C. \( y = \frac{3x}{x-1} \) \((\text{answer})\)
   - D. \( y = \frac{3x^2}{x^2-x} \)
6. Which statement about the graph of \( y = \frac{x^2 - 5x + 6}{x - 3} \) is true?

A. there is a vertical asymptote with the equation \( y = 3 \)
B. there is a horizontal asymptote with the equation \( x = 2 \)
C. there is a point of discontinuity at \((3, 1)\) (answer)
D. there is a point of discontinuity at \((1, 3)\)

7. For the graph of \( y = \frac{3x + 7}{2x + 5} \), determine the equation of the asymptote(s) and the range. SE

Vertical Asymptote: \( x = -\frac{5}{2} \)

Horizontal Asymptote: \( y = \frac{3}{2} \)

8. Determine the coordinates of the point of discontinuity on the graph of \( f(x) = \frac{2x^2 - 15x + 7}{x - 7} \). SE

\((7, 14)\)

9. The graph of the function below can be expressed in the form \( y = \frac{ax}{x^2 - bx - c} \).

Determine the values of \( a, b, \) and \( c \). SE

(Point \((2, -1)\) is on the graph, as shown)

\[
\begin{align*}
y &= \frac{ax}{(x + 4)(x - 5)} \\
y &= \frac{ax}{x^2 - x - 20} \\
-1 &= \frac{a(2)}{(2)^2 - (2) - 20} \\
-1 &= \frac{2a}{-18} \\
a &= 9 \\
y &= \frac{9x}{x^2 - x - 20} \\
a &= 9 \\
b &= -1 \\
c &= -20
\end{align*}
\]
10. The horizontal asymptote of the graph of the function \( g(x) = \frac{-7x + 2}{3x + 2} \) is

A. \( y = 0 \)
B. \( y = -\frac{7}{3} \) (answer)
C. \( y = -\frac{3}{7} \)
D. \( y = -\frac{2}{3} \)

11. Consider the rational function \( f(x) = \frac{ax + 5}{7x - 6} \), where a and b are natural numbers.

There vertical asymptote and the horizontal asymptote of the graph of \( f(x) \), respectively, are

A. \( x = b, y = a \)
B. \( x = \frac{7}{b}, y = -\frac{5}{a} \)
C. \( x = \frac{b}{7}, y = \frac{a}{7} \) (answer)
D. \( x = \frac{b}{7}, y = -\frac{a}{7} \)

12. The graph of the function \( f(x) = \frac{3x - 8}{x - 8} \) has a vertical asymptote with the equation \( x = p \) and a horizontal asymptote with equation \( y = q \).

Place the value of \( p \) in the first box and \( q \) in the second box.

Vertical Asymptote \( x = 8 \)
Horizontal Asym. \( y = 3 \)
13. The equations of seven rational functions and the graphs of these functions are shown. If $a$, $b$, and $c$ are distinct natural numbers, match the set of rational functions to their graphs and explain the reasoning.

<table>
<thead>
<tr>
<th>Function 1</th>
<th>Function 2</th>
<th>Function 3</th>
<th>Function 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y = \frac{1}{x+b}$</td>
<td>$y = \frac{x}{x+b}$</td>
<td>$y = \frac{x+a}{x+b}$</td>
<td>$y = \frac{(x+a)(x+c)}{x+b}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Function 5</th>
<th>Function 6</th>
<th>Function 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y = \frac{(x+a)(x+b)}{x+b}$</td>
<td>$y = \frac{x+a}{(x+b)(x+c)}$</td>
<td>$y = \frac{x+c}{(x+b)(x+c)}$</td>
</tr>
</tbody>
</table>

A. 
B. 
C. 
D. 
E. 
F. 
G.